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The Brumgnach Method for finding the combination of minterms that yielded a specific simplified term.

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Assume that in a problem with four logic variables the variables are assigned the following weights: D=8, C=4, B=2 and A=1.

A minterm is the decimal equivalent of the binary value of the "intersection" (AND combination) of the variables. For example, $\overline{D}\bar{C}BA$ may be referred to as minterm 3 (or m3 for short).

In the design of logic circuits it is desireble to obtain the combination of minterms that would result in a circuit with the smallest number of logic gates and the logic gates having the least number of inputs.

This simplification is done with Boolean Algebra.

Sometimes it may be required to work backwards. Given a simplified term it may be required to find the combination of minterms that yielded that term.

To the eye untrained in Boolean Algebra, it may be quite difficult to determine which minterms were used in the "union" (OR combination) to yield such a term as $\overline{D}\overline{B}$.

The following method makes this task very easy. The method is summarized in the tables shown below. This particular problem is summarized in Case 2.

Examine the term given; in this case $\overline{D}\overline{B}$.

The first minterm is determined by adding the weights of the variables in the term; using the direct (or uncomplemented) form of the variable as a 1 and the complemented form of the variable as a 0. For the B variable: $B = 1$ while $\overline{B} = 0$. In this case, $\overline{D}\overline{B} = 0 + \ 0 = 0$. The first minterm then is m0.

Notice that the variables C and A are missing from the term. These variables have weights of 4 and 1 respectively.

The second minterm is obtained by adding the first minterm to the weight of the first missing variable. In this case: 0+4=4. Therefore, the second minterm is m4.

The third minterm is obtained by adding the first minterm to the weight of the second missing variable. In this case: 0+1=1. Therefore, the second minterm is m1.

The fourth minterm is obtained by adding the first minterm to the weight of the first and the second missing variables. In this case: 0+4+1 =5. Therefore, the fourth minterm is m5.

Then: $\overline{D}\overline{B}$ = m0+m1+m4+m5

Or: $\overline{D}\ \overline{B} = \overline{D}\bar{C}\overline{B}\overline{A} + \overline{D}\bar{C}\overline{B}A + \overline{D}C\overline{B}\overline{A} + \overline{D}C\overline{B}A$

Using Boolean Algebra this can be shown to be true by the following 3 steps ($\bar{x} + x = 1$):

1) Combine m0 and m1 by factoring out $\bar{D}\bar{C}\bar{B}$: $\bar{D}\bar{C}\bar{B}\bar{A} + \bar{D}\bar{C}\bar{B}A = \bar{D}\bar{C}\bar{B}(\bar{A} + A) = \bar{D}\bar{C}\bar{B}.$
2) Combine m4 and m5 by factoring out $\bar{D}C\bar{B}$: $\bar{D}C\bar{B}\bar{A} + \bar{D}C\bar{B}A = \bar{D}C\bar{B}(\bar{A} + A) = \bar{D}C\bar{B}.$
3) Combine the result of the m0 and m1 combination with the result of the m4 and m5 combination: $\bar{D}\bar{C}\bar{B} + \bar{D}C\bar{B} = \bar{D}\bar{B}(\bar{C} + C) = \bar{D}\bar{B}$

The following four cases summarize the above method.

Case 1. One variable missing

		Weight ↓	Determination of minterm ↓	minterm ↓
Term →	$\bar{D}\ B\ A$	3	m3	m3
Missing Variable →	C	4	3+4=7	m7

Then: $\bar{D}BA$ = m3 + m7
Or: $\bar{D}BA = \bar{D}\bar{C}BA + \bar{D}CBA$

Case 2. Two variables missing

		Weight ↓	Determination of minterm ↓	minterm ↓
Term →	$\bar{D}\bar{B}$	0+0=0	m0	m0
Missing Variables →	C	4	0+4=4	m4
	A	1	0+1=1	m1
Combination of missing variables →	CA	4+1=5	0+5=5	m5

Then: CA = m0 + m1 + m4 + m5
Or: $CA = \bar{D}\bar{C}\bar{B}\bar{A} + \bar{D}\bar{C}\bar{B}A + \bar{D}C\bar{B}\bar{A} + \bar{D}C\bar{B}A$

Case 3. Three variables missing

		Weight ↓	Determination of minterm ↓	minterm ↓
Term →	B	2	m2	m2
Missing Variables →	D	8	2+8=10	m10
	C	4	2+4=6	m6
	A	1	2+1=3	m3
Combination of missing variables →	DC	8+4=12	2+12=14	m14
	DA	8+1=9	2+9=11	m11
	CA	4+1=5	2+5=7	m7
	DCA	8+4+1=13	2+13=15	m15

Then: B = m2 + m3 +m6 +m7 + m10 + m11 + m14 + m15
Or: $B = \bar{D}\bar{C}B\bar{A} + \bar{D}\bar{C}BA + \bar{D}CB\bar{A} + \bar{D}CBA + D\bar{C}B\bar{A} + D\bar{C}BA + DCB\bar{A} + DCBA$

Case 4. If the term is a 1 (all variables missing) then all 15 minterms are present:
1=m0+m1+m2+m3+m4+m5+m6+m7+m8+m9+m10+m11+m12+m13+m14+m5.